9.16 As noticed in Exercise 14, $f''(x) = \dfrac{2}{x^3}$, for $x \neq 0$.

Since $f''(x) > 0$ for all $x > 0$, the function is convex on the interval $(0, \infty)$.

Since $f''(x) < 0$ for all $x < 0$, the function is concave on the interval $(-\infty, 0)$.

9.18 (a) For $x > 0$, $f'(x) = \dfrac{1}{2\sqrt{x}}$ and $f''(x) = -\dfrac{1}{4x\sqrt{x}}$. So $f(1) = 1, f'(1) = \frac{1}{2}$ and $f''(1) = -\frac{1}{4}$.
Hence, the Taylor polynomial p_2 of degree 2 for f at 1 is given by

$$p_2(x) = 1 + \tfrac{1}{2}(x-1) - \tfrac{1}{8}(x-1)^2.$$

(b) For $x \neq -\frac{1}{2}$, $f'(x) = -\dfrac{6}{(1+2x)^2}$ and $f''(x) = \dfrac{24}{(1+2x)^3}$. So $f(1) = 1, f'(1) = -\frac{6}{9}$ and $f''(1) = \frac{24}{27}$.
Hence, the Taylor polynomial p_2 of degree 2 for f at 1 is given by

$$p_2(x) = 1 - \tfrac{2}{3}(x-1) + \tfrac{8}{9}(x-1)^2.$$

9.20 (b) As $f''(x) = -\frac{1}{4}x^{-1\frac{1}{2}}$,

$$f^{(3)}(x) = \tfrac{3}{8}x^{-2\frac{1}{2}} = \frac{3}{8x^2\sqrt{x}}.$$

Hence, $f^{(3)}(1) = \frac{3}{8}$ and the Taylor polynomial p_3 of degree 3 for f at 1 is given by

$$p_3(x) = 1 + \tfrac{1}{2}(x-1) - \tfrac{1}{8}(x-1)^2 + \tfrac{1}{16}(x-1)^3.$$

9.21 (a) According to the Arithmetic Rules for limits of functions,

$$\lim_{x\to 0} \frac{x^2}{2x^2+x} = \lim_{x\to 0} \frac{x}{2x+2} = \frac{\lim\limits_{x\to 0} x}{\lim\limits_{x\to 0}[2x+2]} = \frac{0}{2} = 0.$$

(b) Note that for $x \neq 2$,

$$\frac{2x^2-x-6}{3x^2-7x+2} = \frac{2\big(x+1\frac{1}{2}\big)(x-2)}{3\big(x-\frac{1}{3}\big)(x-2)} = \frac{2}{3}\,\frac{x+1\frac{1}{2}}{x-\frac{1}{3}}.$$

So, according to the Arithmetic Rules for limits of functions,

$$\lim_{x\to 2} \frac{2x^2-x-6}{3x^2-7x+2} = \lim_{x\to 2} \frac{2}{3}\,\frac{x+1\frac{1}{2}}{x-\frac{1}{3}} = \frac{7}{5}.$$

9.22 We introduce the differentiable function f on $(-1, 1)$, defined by $f(x) = \sqrt{x+1} - 1 + \frac{1}{2}x$ and the differentiable function g, defined by $g(x) = x$. Then $g' = 1 \neq 0$ and $f(0) = g(0) = 0$. So the weak form of de l'Hôpitals Rule implies that

$$\lim_{x\to 0} \frac{\sqrt{x+1} - 1 + \frac{1}{2}x}{x} = \frac{f'(0)}{g'(0)} = \frac{1}{2\sqrt{0+1}} + \tfrac{1}{2} = 1.$$

9.24 The numerator and denominator of the fraction $\dfrac{1}{2x + \cos x}$ are not equal to zero at $x = 0$. So we may not apply de l'Hôpital's rule.

9.32 Assume that $x > a$.

The function f restricted to the interval $[a, x]$ is continuous and differentiable on the interval (a, x). According to the Mean Value Theorem, there exists a $\tau \in (a, x)$ such that

$$\frac{f(x) - f(a)}{x - a} = f'(\tau) \Longrightarrow f(x) - f(a) = f'(\tau)(x - a) \geq 0 \Longrightarrow f(x) \geq f(a).$$

8.34 Note that

$$x^3 - 7x^2 + 16x - 12 = (x-2)^2(x-3)$$

and

$$x^2 - 4x + 4 = (x-2)^2.$$

By consequence

$$\lim_{x \to 2} \frac{x^3 - 7x^2 + 16x - 12}{x^2 - 4x + 4} = \lim_{x \to 2} \frac{(x-2)^2(x-3)}{(x-2)^2} = \lim_{x \to 2} (x - 3) = -1.$$

9.36 Because the functions h and h' are continuous, the Arithmetic Rules for differentiable functions imply that the function g is continuous. Hence, the restriction of the function g to the interval $[0, 2]$ is continuous and differentiable on the interval $(0, 2)$.

Since, furthermore, $g(0) = g(2)$, Rolle's Theorem implies the existence of a $\tau \in (0, 2)$ such that

$$g'(\tau) = 0 \Longrightarrow h'(\tau) + 3h''(\tau) = 0 \Longrightarrow h'(\tau) = -3h''(\tau).$$

Since

$$0 \leq h''(\tau) \leq \tfrac{1}{3} \Longrightarrow -1 \leq -3h''(\tau) \leq 0,$$

it follows that $-1 \leq h'(\tau) \leq 0$.

Because $h'' \geq 0$, the function h' is increasing. Then the inequality $\tau < 2$ implies that

$$h'(2) \geq h'(\tau) \geq -1.$$

9.37 Since $f(0) = 1$ and $f'(0) = f(0) = 1$, the linear approximation ℓ_0 of f at 0 is given by

$$\ell_0(x) = f(0) + f'(0)x = 1 + x.$$

Further, $f'' = f' = f$. So for $x \neq 0$ the remainder is

$$r(x) = \frac{f''(\tau)}{2} x^2 = \tfrac{1}{2} f(\tau) x^2,$$

where τ is between 0 and x.

11.7 Since we can write the denominator of the fraction

$$\frac{x}{x^2-5x+6}$$

as $x^2-5x+6=(x-2)(x-3)$, we try to find constants A and B such that for all $x\neq 2,3$

$$\begin{aligned}\frac{x}{(x-2)(x-3)}=\frac{A}{x-2}+\frac{B}{x-3} &\iff \frac{x}{(x-2)(x-3)}=\frac{A(x-3)+B(x-2)}{(x-2)(x-3)}\\ &\iff \frac{x}{(x-2)(x-3)}=\frac{(A+B)x-3A-2B}{(x-2)(x-3)}\\ &\iff x=(A+B)x-3A-2B.\end{aligned}$$

If we choose $x=0$, then we find that $-3A-2B=0$. Hence, $x=(A+B)x$ for all $x\neq 2,3$. This leads to $A+B=1$ (as one can see by choosing $x=1$). In other words: A and B are solutions of the system

$$\begin{cases} A+B=1\\ -3A-2B=0\end{cases} \iff \begin{cases} 2A+2B=2\\ -3A-2B=0\end{cases} \iff \begin{cases} A=-2\\ -3A-2B=0\end{cases} \iff \begin{cases} A=-2\\ B=3.\end{cases}$$

Hence,

$$\begin{aligned}\int_0^1 \frac{x}{x^2-5x+6}\,\mathrm{d}x &= \int_0^1\left(\frac{-2}{x-2}+\frac{3}{x-3}\right)\mathrm{d}x=\left[-2\ln|x-2|+3\ln|x-3|\right]_0^1\\ &=3\ln 2+2\ln 2-3\ln 3=\ln\frac{2^5}{3^3}=\ln\tfrac{32}{27}.\end{aligned}$$

11.14 Note that for $x\neq 0,1,-1$

$$\frac{x^5+2x^2+1}{x^3-x}=\frac{x^2(x^3-x)+x^3-x+x+2x^2+1}{x^3-x}=x^2+1+\frac{2x^2+x+1}{x^3-x}.$$

We try to find constants A,B and C satisfying

$$\frac{2x^2+x+1}{x(x-1)(x+1)}=\frac{A}{x}+\frac{B}{x-1}+\frac{C}{x+1}.$$

Hence, for all $x\neq 0,1,-1$

$$2x^2+x+1=A(x^2-1)+Bx(x+1)+Cx(x-1)\iff 2x^2+x+1=(A+B+C)x^2+(B-C)x-A.$$

So A,B and C satisfy the system

$$\begin{cases} A+B+C=2\\ B-C=1\\ -A=1\end{cases}\iff \begin{cases} A=-1\\ B=2\\ C=1.\end{cases}$$

Hence,

$$\begin{aligned}\int_2^3\frac{x^5+2x^2+1}{x^3-x}\,\mathrm{d}x&=\int_2^3\left(x^2+1-\frac{1}{x}+\frac{2}{x-1}+\frac{1}{x+1}\right)\mathrm{d}x\\ &=\left[\tfrac{1}{3}x^3+x-\ln x+2\ln(x-1)+\ln(x+1)\right]_2^3\\ &=9+3-\ln 3+2\ln 2+\ln 4-\tfrac{8}{3}-2+\ln 2-\ln 3=7\tfrac{1}{3}+\ln\tfrac{32}{9}.\end{aligned}$$

11.15 Note that

$$\int_1^{\frac{4}{3}} \sqrt{\frac{x-1}{x^5}}\,\mathrm{d}x = \int_1^{\frac{4}{3}} \frac{1}{x^2}\,\sqrt{1-\frac{1}{x}}\,\mathrm{d}x.$$

We use the Method of Substitution with $\varphi(x) = 1 - \dfrac{1}{x}$. Then $\varphi'(x) = \dfrac{1}{x^2}$. So we obtain

$$\begin{aligned}\int_1^{\frac{4}{3}} \sqrt{\frac{x-1}{x^5}}\,\mathrm{d}x &= \int_1^{\frac{4}{3}} \frac{1}{x^2}\,\sqrt{1-\frac{1}{x}}\,\mathrm{d}x = \int_1^{\frac{4}{3}} \sqrt{\varphi(x)}\;\varphi'(x)\,\mathrm{d}x = \tfrac{2}{3}\left[\varphi(x)^{1\frac{1}{2}}\right]_1^{\frac{4}{3}} \\ &= \tfrac{2}{3}\varphi(\tfrac{4}{3})^{1\frac{1}{2}} - \tfrac{2}{3}\varphi(1)^{1\frac{1}{2}} = \tfrac{1}{12}.\end{aligned}$$