1.1 (a) true: the numbers $4, 3$ and 2 are elements of the set A (the order of the elements is irrelevant).

(b) not true: there is no $n \in \mathbb{N}$ such that $3 = 2n$.

(c) true: since each element of A is a natural number smaller than 6, each element of the set A is contained in the set C.

(d) not true: the set A doesn't contain the **set** $\{2\}$; however, it contains the number 2.

(e) not true: the set C contains the number 3 which is not contained in the set B.

(f) true: by choosing $n = 1, 2, 3, 4$, respectively, one shows that the numbers $2, 4, 6$ and 8 are contained in the set B.

(g) true: each element of the set C is contained in the set A.

(h) true: since $A \subset C$ and $C \subset A$, $A = C$.

1.2 $A \cup B = \{1, 2, 5, 7, 10, 11, 14\}$

$A \cap B = \{7\}$

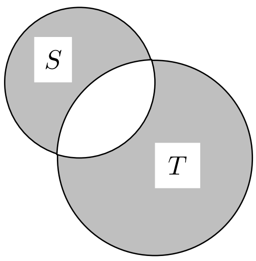
$A \setminus B = \{1, 5, 10\}$

$B \setminus A = \{2, 11, 14\}$

$\big(A \setminus B\big) \cup B = \{1, 2, 5, 7, 10, 11, 14\}$

$\big(A \setminus B\big) \cup \big(A \cap B\big) = \{1, 5, 7, 10\}$.

1.4 (a) A Venn diagram of the set $S \,\Delta\, T$ is the gray area below:



(b) According to the definition,

$$S \,\Delta\, S = \big(S \setminus S\big) \cup \big(S \setminus S\big) = \phi \cup \phi = \phi.$$

(c) According to the definition,

$$S \,\Delta\, \phi = \big(S \setminus \phi\big) \cup \big(\phi \setminus S\big) = S \cup \phi = S.$$

1.7 (a) If $x \in [0, b]$, then x can be written in the form $x = \dfrac{x}{b} \cdot b$, whereas

$$0 \le x \le b \Longrightarrow 0 \le \frac{x}{b} \le 1.$$

So we can choose $t = \dfrac{x}{b}$.

We obtain the midpoint of the interval $[0, b]$ if we choose $t = \frac{1}{2}$.

(b) Note that $b - a > 0$. If $x \in [a, b]$, then

$$a \le x \le b \Longleftrightarrow 0 \le x - a \le b - a \Longrightarrow x - a \in [0, b - a].$$

So, according to part (a), there exists some $t \in [0,1]$ such that

$$x - a = t(b-a) \iff x = a + t(b-a) \iff x = (1-t)a + tb.$$

(c) If $t \in \{0,1\}$, then, obviously $(1-t)a + tb \in [a,b]$.
If $0 < t < 1$, then

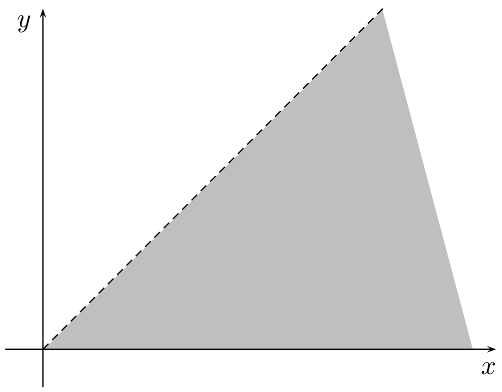
$$(1-t)a + tb < (1-t)b + tb = b$$

and
$$(1-t)a + tb > (1-t)a + ta = a.$$

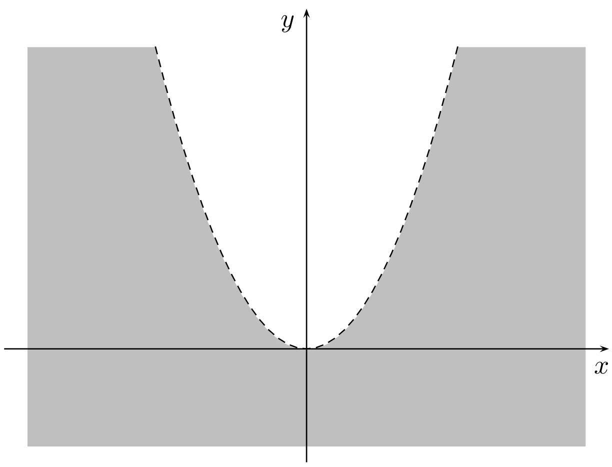
Hence, $(1-t)a + tb \in [a,b]$.

(d) In a similar way, one can prove that $x \in (a,b)$ if and only if $x = (1-t)a + tb$ for some $t \in (0,1)$.

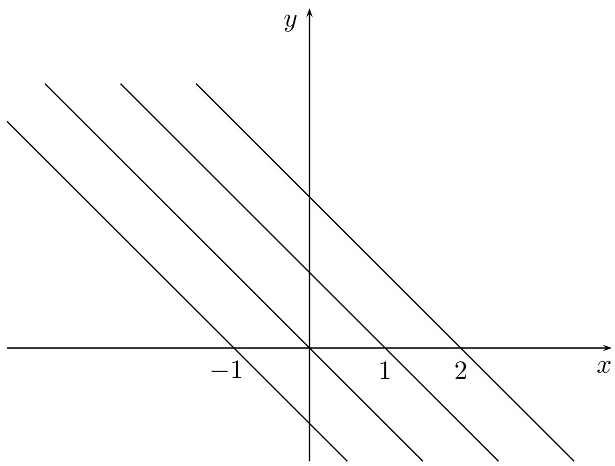
1.8 (a) The set $\{(x,y) |\ x > y\}$:



(b) The set $\{(x,y) |\ y < x^2\}$:

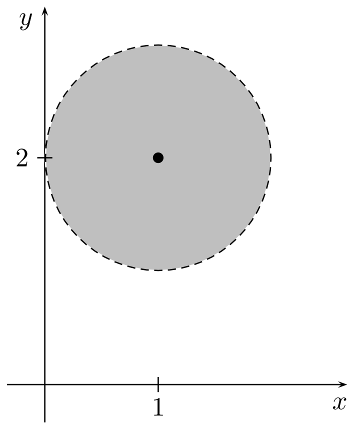


(c) Note that $x + y = k \iff y = k - x$, where $k \in \mathbf{Z}$. Hence, the set $\{(x,y) |\ x + y \text{ is an integer}\}$ consists of an infinite number of lines $y = k - x$, where $k \in \mathbf{Z}$.

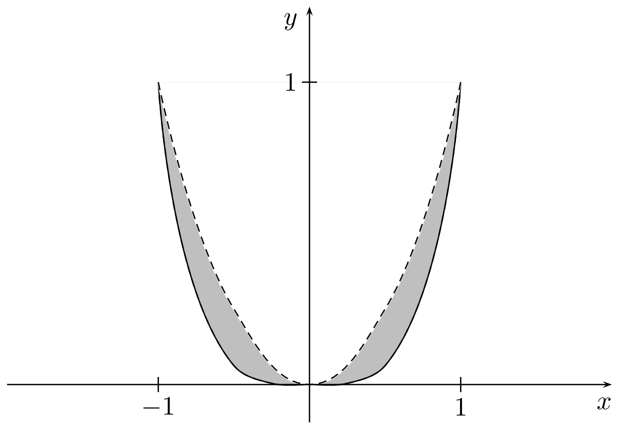


(d) The set $(x-1)^2+(y-2)^2=1$ represents a circle with center $(1,2)$ and radius 1.

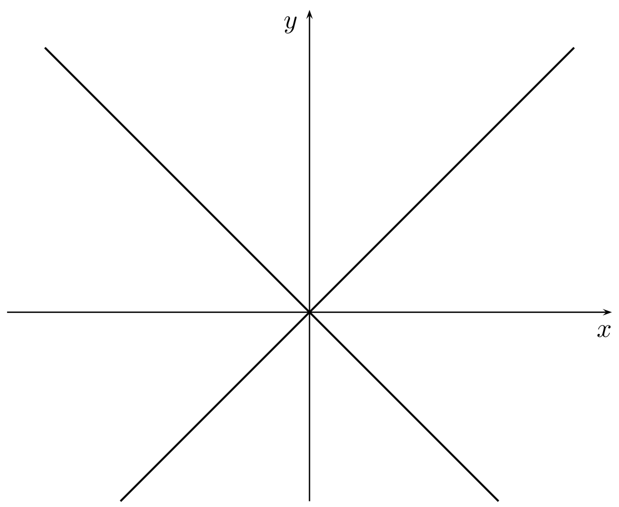
The set $(x-1)^2+(y-2)^2<1$:



(e) Note that $x^4 < x^2 \iff x^2(x^2-1)<0 \iff -1<x<0$ or $0<x<1$. The set $\{(x,y)|\ x^4<y<x^2\}$ is the set of points 'between' the parabola $y=x^2$ and the parabola-like curve $y=x^4$:



(f) Since $x^2=y^2 \iff y=\pm x$, the set $\{(x,y)|\ x^2=y^2\}$ consists of the two lines $y=x$ and $y=-x$:



1.25 For $x \neq -1$ and $g(x) \neq -1$,

$$(g \circ g)(x) = g\big(g(x)\big) = g\Big(\frac{1-x}{1+x}\Big) = \frac{1-g(x)}{1+g(x)} = \frac{1-\dfrac{1-x}{1+x}}{1+\dfrac{1-x}{1+x}} = \frac{1+x-(1-x)}{1+x+(1-x)} = \frac{2x}{2} = x.$$

As for $x \neq -1$,

$$g(x) = -1 \Longleftrightarrow \frac{1-x}{1+x} = -1 \Longleftrightarrow 1-x = -1-x \Longleftrightarrow 2 = 0,$$

the domain of $g \circ g$ is the set $\mathbb{R} \setminus \{-1\}$.

1.51 (a) $2x^4 - 32 = 2(x^4-16) = 2(x^2-4)(x^2+4) = 2(x-2)(x+2)(x^2+4)$.

(b) $x^2 - y^2 + 5x + 5y = (x-y)(x+y) + 5(x+y) = (x+y)(x-y+5)$.

1.52 (a) By rewriting the fractions as fractions with x^2y^2 as the common denominator, we obtain for $x \neq 0$ and $y \neq 0$,

$$\frac{2+x}{x^2y} + \frac{1-y}{xy^2} - \frac{2y}{x^2y^2} = \frac{(2+x)y}{x^2y^2} + \frac{(1-y)x}{x^2y^2} - \frac{2y}{x^2y^2} = \frac{2y+xy+x-xy-2y}{x^2y^2}$$
$$= \frac{x}{x^2y^2} = \frac{1}{xy^2}.$$

(b) By rewriting the fractions as fractions with x^2-y^2 as the common denominator, we obtain for $x \neq \pm y$,

$$\frac{x-y}{x+y} - \frac{x}{x-y} + \frac{3xy}{x^2-y^2} = \frac{(x-y)^2}{x^2-y^2} - \frac{x(x+y)}{x^2-y^2} + \frac{3xy}{x^2-y^2}$$
$$= \frac{x^2-2xy+y^2-x^2-xy+3xy}{x^2-y^2} = \frac{y^2}{x^2-y^2}.$$

(c) For $x > 0$,

$$\frac{x\big(\sqrt{x}+1\big)}{\sqrt{x}} = \sqrt{x}\big(\sqrt{x}+1\big) = x + \sqrt{x}.$$

1.53 (b) First we observe that

$$\frac{x}{2} \geq 1 + \frac{4}{x} \Longleftrightarrow \frac{x^2}{2x} - \frac{2x}{2x} - \frac{8}{2x} \geq 0 \Longleftrightarrow \frac{x^2-2x-8}{2x} \geq 0 \Longleftrightarrow \frac{(x+2)(x-4)}{2x} \geq 0.$$

Next we construct a *sign diagram* for the expressions $(x+2)(x-4)$, $2x$ and $\dfrac{(x+2)(x-4)}{2x}$:

+	0	−	−	−	−	−	0	+	$(x+2)(x-4)$
	-2						4		
−	−	−	0	+	+	+	+	+	$2x$
			0						
−	0	+	×	−	−	−	0	+	$\dfrac{(x+2)(x-4)}{2x}$
	-2		0				4		

So the original inequality is satisfied for $-2 \leq x < 0$ and for $x \geq 4$.

(c) Since $(x-1)^{100} \geq 0$ for all x, the sign of the expressions $(x-1)^{100}(x+1)$ is determined by the factor $x+1$. Hence

$$(x-1)^{100}(x+1) > 0 \Longleftrightarrow x+1 > 0 \text{ and } x \neq 1 \Longleftrightarrow -1 < x < 1 \text{ or } x > 1.$$

(d) First we solve the inequalities $3x - 1 < 5x + 3$ and $5x + 3 \leq 2x + 15$ separately. Since

$$3x - 1 < 5x + 3 \iff -4 < 2x \iff x > -2$$

and

$$5x + 3 \leq 2x + 15 \iff 3x \leq 12 \iff x \leq 4,$$

the inequalities are satisfied if $-2 < x \leq 4$.