12.13 Since

$$\text{proj}_W(\underline{e}_1) = (\underline{e}_1 \cdot \underline{w}_1)\,\underline{w}_1 + (\underline{e}_1 \cdot \underline{w}_2)\,\underline{w}_2 = 0 + (-\tfrac{4}{5})\begin{bmatrix} -\frac{4}{5} \\ 0 \\ \frac{3}{5} \end{bmatrix} = \tfrac{1}{25}\begin{bmatrix} 16 \\ 0 \\ -12 \end{bmatrix},$$

$$\text{proj}_W(\underline{e}_2) = (\underline{e}_2 \cdot \underline{w}_1)\,\underline{w}_1 + (\underline{e}_2 \cdot \underline{w}_2)\,\underline{w}_2 = 1 \cdot \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + 0 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

and

$$\text{proj}_W(\underline{e}_3) = (\underline{e}_3 \cdot \underline{w}_1)\,\underline{w}_1 + (\underline{e}_3 \cdot \underline{w}_2)\,\underline{w}_2 = 0 + \tfrac{3}{5}\begin{bmatrix} -\frac{4}{5} \\ 0 \\ \frac{3}{5} \end{bmatrix} = \tfrac{1}{25}\begin{bmatrix} -12 \\ 0 \\ 9 \end{bmatrix},$$

the standard matrix of the orthogonal projection proj_W is given by

$$A = \tfrac{1}{25}\begin{bmatrix} 16 & 0 & -12 \\ 0 & 25 & 0 \\ -12 & 0 & 9 \end{bmatrix}.$$

12.16 Note that the vectors $\underline{w}_1$ and $\underline{w}_2$ are orthogonal. We normalize these vectors and we obtain

$$\underline{v}_1 = \frac{\underline{w}_1}{\|\underline{w}_1\|} = \frac{1}{\sqrt{30}}\begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix}$$

and

$$\underline{v}_2 = \frac{\underline{w}_2}{\|\underline{w}_2\|} = \frac{1}{\sqrt{6}}\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}.$$

Next we determine

$$\text{proj}_W(\underline{u}) = \big(\underline{u} \cdot \underline{v}_1\big)\,\underline{v}_1 + \big(\underline{u} \cdot \underline{v}_2\big)\,\underline{v}_2 = \tfrac{1}{2}\begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix} + (-\tfrac{7}{2})\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} -1 \\ -8 \\ 4 \end{bmatrix}.$$

Finally, the distance between the vector $\underline{u}$ and W is

$$\|\underline{u} - \text{proj}_W(\underline{u})\| = \| \begin{bmatrix} 0 \\ 3 \\ 6 \end{bmatrix} \| = \sqrt{9+36} = \sqrt{45} = 3\sqrt{5}.$$

12.17 As $\underline{v}_1$ and $\underline{v}_2$ are orthogonal, we let $\underline{w}_1 = \underline{v}_1$ and $\underline{w}_2 = \underline{v}_2$.

Next we determine

$$\underline{w}_3 = \underline{v}_3 - \frac{\underline{v}_3 \cdot \underline{w}_1}{\|\underline{w}_1\|^2}\underline{w}_1 - \frac{\underline{v}_3 \cdot \underline{w}_2}{\|\underline{w}_2\|^2}\underline{w}_2 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} - \tfrac{4}{3}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \tfrac{1}{2}\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} = \tfrac{1}{6}\begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}.$$

By normalizing these vectors we obtain the orthonormal basis of $\mathbb{R}^3$ formed by the vectors

$$\frac{1}{\sqrt{3}}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \frac{1}{\sqrt{2}}\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \quad \text{and} \quad \frac{1}{\sqrt{6}}\begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}.$$

12.18 We let

$$\underline{w}_1 = \underline{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

and

$$\underline{w}_2 = \underline{v}_2 - \frac{\underline{v}_2 \cdot \underline{w}_1}{\| \underline{w}_1 \|^2} \underline{w}_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} - \tfrac{3}{4} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \tfrac{1}{4} \begin{bmatrix} -3 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

and

$$\underline{w}_3 = \underline{v}_3 - \frac{\underline{v}_3 \cdot \underline{w}_1}{\| \underline{w}_1 \|^2} \underline{w}_1 - \frac{\underline{v}_3 \cdot \underline{w}_2}{\| \underline{w}_2 \|^2} \underline{w}_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} - \tfrac{2}{4} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} - \tfrac{\frac{1}{2}}{\frac{3}{4}} \cdot \tfrac{1}{4} \begin{bmatrix} -3 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} - \tfrac{1}{2} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} - \tfrac{1}{6} \begin{bmatrix} -3 \\ 1 \\ 1 \\ 1 \end{bmatrix} = \tfrac{1}{3} \begin{bmatrix} 0 \\ -2 \\ 1 \\ 1 \end{bmatrix}.$$