6.1 Obviously,

$$(A+B)_{ij} = a_{ij} + b_{ij} = b_{ij} + a_{ij} = (B+A)_{ij}.$$

6.2 (a) The matrix BA is not defined: $4 \times \boxed{5 \cdot 4} \times 5$.

(b) The matrix $AC + D$ is a 4×2 matrix: $4 \times \boxed{5 \cdot 5} \times 2 + 4 \times 2 \to 4 \times 2$.

(c) The matrix $AE + B$ is not defined: $4 \times \boxed{5 \cdot 5} \times 4 + 4 \times 5 \to 4 \times 4 + 4 \times 5$.

(d) The matrix $AB + B$ is not defined: $4 \times \boxed{5 \cdot 4} \times 5$.

(e) The matrix $E(A+B)$ is a 5×5 matrix: $5 \times \boxed{4 \cdot 4} \times 5 \to 5 \times 5$.

(f) The matrix AC is a 4×2 matrix: $4 \times \boxed{5 \cdot 5} \times 2 \to 4 \times 2$.

The matrix $E(AC)$ is a 5×2 matrix: $5 \times \boxed{4 \cdot 4} \times 2 \to 5 \times 2$.

6.3 (a) The first row of the 3×3 matrix AB is

$$\begin{bmatrix} 67 & 41 & 41 \\ * & * & * \\ * & * & * \end{bmatrix}.$$

(b) The first column of the 3×3 matrix BA is

$$\begin{bmatrix} 6 & * & * \\ 6 & * & * \\ 63 & * & * \end{bmatrix}.$$

(c) The entry at the position $(3,1)$ of the 3×3 matrix AA is

$$0 \cdot 3 + 4 \cdot 6 + 9 \cdot 0 = 24.$$

6.4 The entry at the position (i,j) of the matrix $A(B+C)$ is equal to

$$\begin{aligned} \big((A(B+C)\big)_{ij} &= \sum_k a_{ik}(B+C)_{kj} = \sum_k a_{ik}\big(b_{kj} + c_{kj}\big) = \sum_k \big(a_{ik}b_{kj} + a_{ik}c_{kj}\big) \\ &= \sum_k a_{ik}b_{kj} + \sum_k a_{ik}c_{kj} = (AB)_{ij} + (AC)_{ij} = (AB + AC)_{ij}, \end{aligned}$$

which is the entry at the position (i,j) of the matrix $AB + AC$.

6.6 (a)

$$D + E = \begin{bmatrix} 7 & 6 & 5 \\ -2 & 1 & 3 \\ 7 & 3 & 8 \end{bmatrix}.$$

(b) The sum of a 2×2 matrix and a 2×3 matrix is not defined.

(c)

$$5A = \begin{bmatrix} 15 & 0 \\ -5 & 10 \\ 5 & 5 \end{bmatrix}.$$

(d)

$$-3(D+2E) = -3\begin{bmatrix} 13 & 7 & 8 \\ -3 & 2 & 5 \\ 11 & 4 & 11 \end{bmatrix} = \begin{bmatrix} -39 & -21 & -24 \\ 9 & -6 & -15 \\ -33 & -12 & -33 \end{bmatrix}.$$

(e)

$$D^T - E^T = (D-E)^T = \begin{bmatrix} -5 & 0 & -1 \\ 4 & -1 & 1 \\ -1 & -1 & 2 \end{bmatrix}.$$

(f)

$$C^T - 4A = \begin{bmatrix} -11 & 3 \\ 8 & -7 \\ -2 & 1 \end{bmatrix}.$$

(g) Note that B^T is a 2×2 matrix and that $5C^T$ is a 3×2 matrix. Hence the matrix $B^T + 5C^T$ is not defined.

(h)

$$\left(2E^T - 3D^T\right)^T = 2E - 3D == \begin{bmatrix} 9 & -13 & 0 \\ 1 & 2 & 1 \\ -1 & -4 & -9 \end{bmatrix}.$$

6.7 (a) As a product of a 3×2 matrix and a 2×2 matrix, AB is a 3×2 matrix and

$$AB = \begin{bmatrix} 12 & -3 \\ -4 & 5 \\ 4 & 1 \end{bmatrix}.$$

(b) The product of a 2×2 matrix and a 3×2 matrix is not defined. So the matrix BA is not defined.

(c) As a product of a 2×3 matrix and a 3×2 matrix, CC^T is a 2×2 matrix and

$$CC^T = \begin{bmatrix} 21 & 17 \\ 17 & 35 \end{bmatrix}.$$

(d) As a product of a 3×3 matrix and a 3×2 matrix, DA is a 3×2 matrix and

$$(DA)^T = \begin{bmatrix} 0 & 12 \\ -2 & 1 \\ 12 & 9 \end{bmatrix}^T = \begin{bmatrix} 0 & -2 & 12 \\ 12 & 1 & 9 \end{bmatrix}.$$

6.11 Note that AA^T is an $m \times m$ matrix and

$$\operatorname{tr}(AA^T) = (AA^T)_{11} + (AA^T)_{22} + \cdots + (AA^T)_{mm}.$$

For every i with $1 \leq i \leq m$

$$(AA^T)_{ii} = \sum_{j=1}^{n} (A)_{ij}(A^T)_{ji} = \sum_{j=1}^{n} a_{ij}a_{ij} = \sum_{j=1}^{n} a_{ij}^2.$$

Hence,

$$\operatorname{tr}(AA^T) = \sum_{i=1}^{m} (AA^T)_{ii} = \sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}^2.$$